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SOME SOLUTIONS OF KARMAN'S EQUATION DESCRIBING THE TRANSONIC FLOW PAST A CORNER POINT ON A PROFILE WITH A CURVILINEAR GENERATRIX†

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Transonic flows in the neighbourhood of a corner point on a profile are investigated in a class of self-similar solutions of Karman's equation. The corner point is formed by the intersection of two smooth curves, the tangents to which make a convex angle. The generatrix, which lies in the subsonic part of the flow is assumed to be curvilinear and to vary according to a power law. Values of the self-similarity index are found for which transonic flows are possible either with a free streamline or with a rarefaction wave.

Transonic flows are investigated in the neighbourhood of a corner point on a profile which is formed by the intersection of two smooth curves AO and OB, the tangents to which form a convex angle (Figs 1 and 2). It is assumed that the subsonic flow of an ideal gas which is incident along AO is vortex-free and iso-energetic and that a sonic line emerges from the point O. In this case all the characteristic features of a flow in a certain neighbourhood of the point O are described by Karman's equation which admits of a class of self-similar solutions [1, 2].

The problem of the transonic flow past a corner point on a profile with a rarefaction wave was first formulated by Vaglio-Laurin when studying the flow past a blunt body with a detached shock wave. Its solution in the case of a rectilinear generatrix AO was found using numerical methods [3] in the class of self-similar functions with self-similarity index $n = 5/4$. The Vaglio-Laurin solution was obtained in a closed parametric form in [4]. If the generatrix AO is rectilinear and $n = 6/5$, then, as shown in [5], a self-similar solution can be obtained which describes a flow with a free stream-line. The velocity in the latter is equal to the velocity of sound.

When $n = 5/4$ and $n = 6/5$, account is taken of the curvature of AO using the following approximations [3, 6–8]. If the curvature of AO is constant, it is taken account of in the Vaglio-Laurin solution using the second approximation, which is responsible for the non-linearity of Karman's equation. However, when $n = 3/2$, a further solution with a rarefaction wave, which is analogous to a Vaglio-Laurin solution, has been found [9]. This solution gives the flow past a corner point with a generatrix AO which has a constant curvature.

Self-similar solutions are found below which describe locally transonic flows in the neighbourhood of a corner point on a profile with a curvilinear generatrix AO which varies according to a power law. It is shown that, apart from the solutions when $n = 5/4$, $n = 6/5$, and $n = 3/2$, there are further solutions with $n \in (1, 2)$ which describe flows with a rarefaction wave (solutions of the Vaglio-Laurin type) as well as flows with a free stream-line (Figs 1 and 2) which are due to the curvilinear property of the generatrix AO.

1. Let us introduce a Cartesian system of coordinates (x, y), where the negative part of the x-axis coincides with the tangent to the generatrix AO at the point O, and a system of Mises coordinates (x, ψ) (ψ is the stream function). The origin of both systems of coordinates is at the point O. We denote by q_x and q_y the velocity components along the x and y-axes respectively, ρ is the density, p is the

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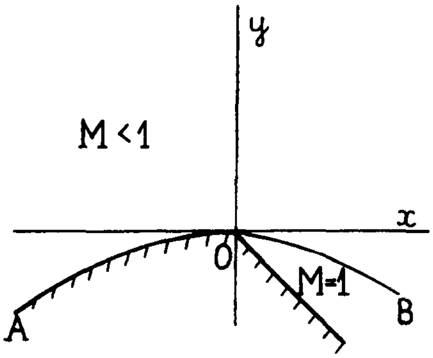

Fig. 1.

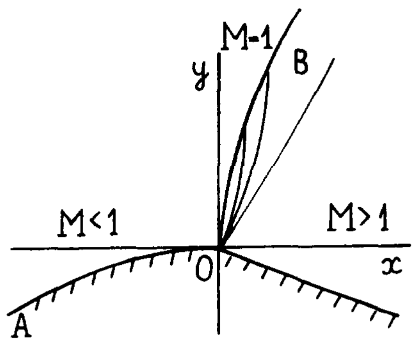

Fig. 2.

pressure and κ is the ratio of the specific heat capacities. The thermodynamic variables are related by the equation of state of an ideal gas. Below, all the flow parameters and the system of Euler equations are assumed to be dimensionless. Their critical values are taken as the characteristic magnitudes.

The components of the perturbed velocity υ_x and υ_y are equal to $\upsilon_x = q_x - 1$, $\upsilon_y = q_x$. We denote the neighbourhood of the corner point O, at which $|\upsilon_x| \ll 1$ and $|\upsilon_y| \gg 1$, by G. In domain G, the system of Euler equations in Mises variables can be simplified and, in the first approximation, can be represented in the form

$$-(1+\kappa)\upsilon_x \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial \psi} = 0, \quad \frac{\partial \upsilon_y}{\partial x} - \frac{\partial \upsilon_x}{\partial \psi} = 0 \tag{1.1}$$

The problem of the transonic flow past a corner point on a profile with a generatrix AO, which varies according to the power law

$$y = -\frac{\beta^{3-3/n}}{4-3/n}\mathcal{B}(-x)^{4-3/n} + \ldots, \quad x \leq 0, \ \mathcal{B} > 0; \ n > 1, \ \beta = (1+\kappa)^{-1/3} \tag{1.2}$$

will be studied using Eqs (1.1).

Depending on the global problem of the flow past a profile, either flow with a rarefaction wave or flow with a free stream-line may be obtained.

Karman's equations (1.1) possess a class of self-similar solutions. For the perturbed velocity potential φ and its components, we have [1, 2]

$$\varphi = \psi^{3n-2}\Phi(\xi), \ \xi = \beta x / \psi^n, \upsilon_x = \beta\psi^{2n-2} f(\xi), \ \upsilon_y = \psi^{3n-3} g(\xi) \tag{1.3}$$

The self-similar function $f(\xi) = d\Phi/d\xi$ satisfies the ordinary differential equation [10]

$$(f - n^2\xi^2)\frac{d^2 f}{d\xi^2} + \left(\frac{df}{d\xi}\right)^2 + n(3n-5)\xi\frac{df}{d\xi} - 2(2n-3)(n-1)f = 0 \tag{1.4}$$

On changing in (1) to the variables

$$F = \xi^{-2} f, \ \Psi = \xi dF / d\xi \tag{1.5}$$

we obtain the first-order differential equation [10]

$$\frac{d\Psi}{dF} = \frac{-6F - 5n\Psi + 6F^2 + 7F\Psi + \Psi^2}{(n^2 - F)\Psi} \tag{1.6}$$

For the function $g(\xi)$ we have

$$g = \frac{1}{3(n-1)}\left[(f - n^2\xi^2)\frac{df}{d\xi} + 2n(n-1)\xi f\right] = \frac{\xi^3}{3(n-1)}[2F(F-n) + (F-n^2)\Psi] \tag{1.7}$$

We shall denote the curves which are determined from the equations

$$2F(F-n) + (F-n^2)\Psi_V = 0$$

$$-6F - 5n\Psi_P + 6F^2 + 7F\Psi_P + \Psi_P^2 = 0$$

by V and P, respectively. The vertical component of the velocity changes sign at the intersection of curve V with the integral curves. The integral curves have a zero slope at points of the curve P.

The solvability of problems in transonic gas dynamics is often conveniently investigated in the hodograph plane. If, in the system of equations (1.1), one changes to a single equation for ψ and to the independent variables $u = \beta^{-1}\upsilon_x$, $\upsilon = \upsilon_y$, then, as a result, the Tricomi equation [1, 2] is obtained. The latter possesses a class of self-similar solutions which correspond to the solutions (1.3) in the physical plane [1]

$$\psi_j = \rho^j \chi_j(z), \quad \rho^2 = \upsilon^2 - \frac{4}{9}u^3, \quad z = \frac{\upsilon^2}{\rho^2}, \quad j = \frac{1}{6(n-2)} \tag{1.8}$$

The function $\chi_j(z)$ satisfies the hypergeometric equation [7]

$$z(1-z)\frac{d^2\chi_j}{dz^2} + \left(\frac{1}{2} - \frac{7}{6}z\right)\frac{d\chi_j}{dz} + j\left(j + \frac{1}{6}\right)\chi_j = 0 \tag{1.9}$$

2. Let us study the behaviour of the integral curves of Eq. (1.6). They have four singular points $A(0,0), B(0,1), C(n^2, -n(n+1)), D(n^2, -6n(n-1))$ in the finite part of the (F, Ψ)-plane and three singular points E, Q and G at infinity [5, 11]. The behaviour of the integral curves is conveniently considered on a unit Poincaré hemisphere projected onto the (F, Ψ)-plane (Figs 3 and 4). As a result, each of the above-mentioned points at infinity is split into two identical points which lie symmetrically on the equator with respect to the centre of the circle. We denote those points at infinity into which the curves enter or from which they depart, when $\Psi > 0$, by E^*, Q^* and G^* (and, when $\Psi > 0$, by E, Q and G).

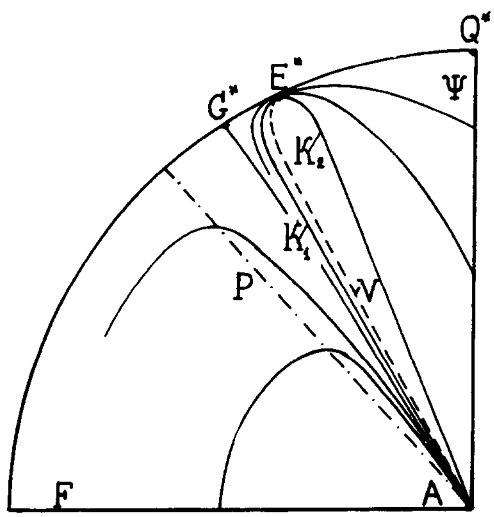

Fig. 3.

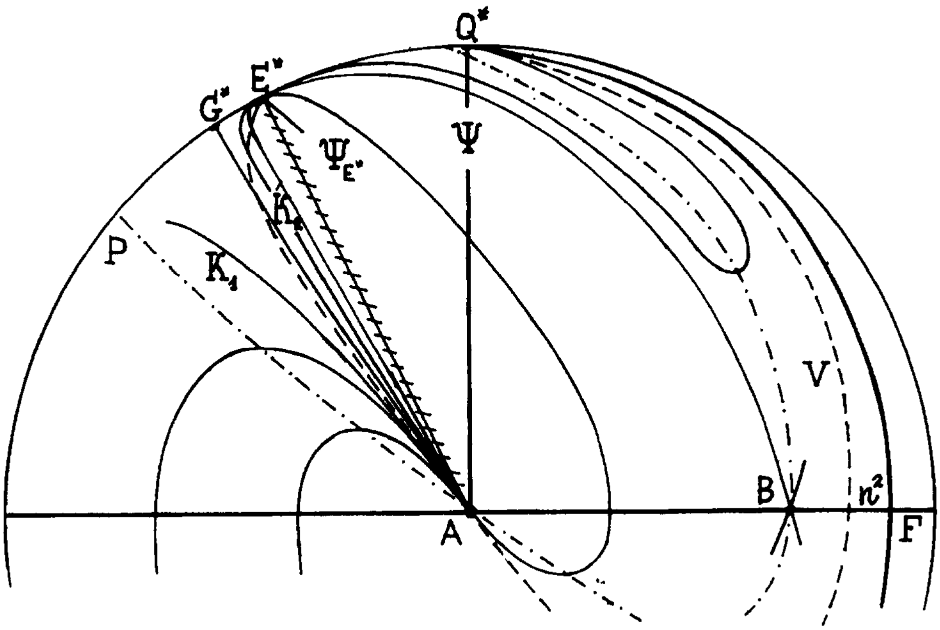

Fig. 4.

Point A is a node and, in the physical plane, corresponds to $\psi = 0, x \neq 0$. When $(-\xi) \to \infty$ in the neighbourhood of the x-axis, the asymptotic behaviour of $\Phi(\xi)$ has the form [1]

$$\Phi(\xi) = A_0(-\xi)^{3-2/n} + B_0(-\xi)^{3-3/n} + \ldots \tag{2.1}$$

In the (F, Ψ)-plane, the integral curves

$$\left(\frac{2}{n}F + \Psi\right) = C_A\left|\frac{3}{n}F + \Psi\right|^{3/2} + \ldots \tag{2.2}$$

correspond to them.

The arbitrary constants A_0 and B_0 in (2.1) are related to C_A in (2.2) and $\mathcal{B}$ in (1.2) by the relation [11]

$$C_A = \frac{3(n-1)n}{(3n-2)^{3/2}}\frac{B_0}{|A_0|^{3/2}}, \quad \mathcal{B} = B_0$$

If $B_0 = 0, A_0 \neq 0$ then $C_A = 0$. It follows that $v_x \neq 0$ and $v_y = 0$. In the physical plane, this corresponds to the flow past a profile with a rectilinear generatrix $y = 0, x < 0$. We shall call the integral curve (2.2) with $C_A = 0$ symmetric and denote it by K_1. We shall call the integral curve (2.2) in the case when $B_0 \neq 0, A_0 = 0$ $(C_A = \infty)$ antisymmetric and denote it by K_2. When $x > 0$, it corresponds to a flow with a free streamline in which the velocity is equal to the velocity of sound.

The singular point B is a saddle point for all values of $n > 1$. We denote the integral curve, which passes through point B with a negative slope and corresponds to flow of the Prandtl–Mayer type, by K_3.

The singular $G - G^*$ is a saddle point. It can only be reached along the integral curve [10]

$$\Psi_G = -\frac{3}{2}F + \frac{9}{10}(n-2)\left(n - \frac{4}{3}\right) + \ldots, \quad |F| \to \infty$$

In the physical plane, the point $G - G^*$ corresponds to the y-axis on which the velocity of sound is attained. Here, the acceleration becomes infinite.

The singular point $E - E^*$ is a node. In the neighbourhood of this point the behaviour of the integral curves can be represented in the form

$$\Psi = -2F + C_{E^*}|F|^{1/2} + 4(n-1)(n-\tfrac{3}{2}) + \ldots, \; |F| \to \infty. \tag{2.3}$$

If the integral curve $f = f(\xi)$ of Eq. (1.4) smoothly traverses the $\xi = 0$ axis when $f < 0$, then, in the neighbourhood of the point $\xi = 0$, two integral curves of the family (2.3) correspond to it when $F \to -\infty$ with $C_{E^*} = \alpha$ and $C_{E^*} = -\alpha$, where α is a certain non-zero constant.

In the physical plane, the point $E - E^*$ corresponds to the y-axis. When $C_{E^*} = 0$ and $F < 0$, we denote the curve of the node by Ψ_{E^*}.

The point $Q - Q^*$ is a node and is arrived at by moving along the limiting line $|\Psi| \to \infty$, $F = n^2 (f = n^2\xi^2)$.

On passing through the point $Q - Q^*$, the magnitude of ξ reaches an extremum. This means that the physical plane doubly overlaps. On approaching the point $Q - Q^*$, the acceleration of the flow tends to infinity.

Points C and D are not used in the investigation and we shall therefore not dwell on them.

Equation (1.6) admits of solutions in the form of linear functions which help in understanding the behaviour of the integral curves as a function of the parameter n and, in fact: $\Psi = -\tfrac{3}{2}F$ when $n = 2$ (curve K_2), $\Psi = -2F$ when $n = \tfrac{3}{2}$ (curve K_2), $\Psi = -\tfrac{3}{2}F$ when $n = \tfrac{4}{3}$ (curve K_1), $\Psi = -2F$ when $n = 1$ (curve K_1) and $\Psi = -2F - 2$ when $n = 2$ (curve K_3).

It can also be easily shown that, when $F < 0$, the inequalities

$$-2 < \frac{d\Psi}{dF}(F,-2F) < 0, \; n \in \left(\frac{3}{2}, 2\right) \tag{2.4}$$

$$\frac{d\Psi}{dF}(F,-2F) < -2, \; n \in \left(1, \frac{3}{2}\right)$$

$$\frac{d\Psi_V}{dF} < \frac{d\Psi}{dF}(F,\Psi_V) < 0, \; n > 1$$

hold on the line $\Psi = -2F$ and the curve V.

When $F < 0$, the curve K_1 departs from point A, located below curve V for all values of $n > 1$. It follows from the inequalities (2.4) that the curves K_1 and V do not intersect when $F > 0$. On the other hand, each integral curve of the family (2.2) with $C_A > 0$, which is located between curves K_1 and K_2 when $F < 0$ intersects the curve V and, moreover, just once.

It follows from the first equality of (1.5) that intersection of the $F = 0$ axis by an integral curve denotes passage through the velocity of sound in the physical plane.

3. We will first consider the problem of the transonic flow past a corner point with a free stream-line. It is formulated as follows. In the domain G, it is required to find a smooth solution of Karman's equations (1.1) which satisfies a no-flow condition on the generatrix AO and the condition of the constancy of the pressure on the free stream-line OB (Fig. 1). To a first approximation, these conditions have the form

$$\upsilon_y \to \beta^{3-3/n}\mathfrak{B}(-x)^{3-3/n}, \; \psi \to 0, \; x < 0 \tag{3.1}$$

$$\upsilon_x \to 0, \; \psi \to 0, \; x > 0 \tag{3.2}$$

The vertical component of the velocity υ_y in the neighbourhood of AO is positive while it is negative in the neighbourhood of OB. If the generatrix AO is rectilinear ($\upsilon_y = 0$ when $\psi = 0$ and $x < 0$), the solution of problem (1.1), (3.1), (3.2) is given in the (F, Ψ)-plane by the functions (Fig. 3)

$$\Psi^{(\pm)} = -5F\frac{5(3-2F) \pm \sqrt{3(3-2F)}}{36-25F}, \; -\infty < F < 0$$

The curve $\Psi^{(-)}$, being the curve K_1, departs from point A and arrives at the point $E^*(C_{E^*} = -\sqrt{(6)}/5)$. At point E^*, it passes into the curve $\Psi^{(+)}$ which is the curve $K_2(C_{E^*} = -\sqrt{(6)}/5)$. The curve Ψ_{E^*} $(C_{E^*} = 0)$ lies between the curves K_1 and K_2. All of the integral curves of the family (2.2), which are located between the curves K_1 and Ψ_{E^*}, arrive at the point E^*. Afterwards, having been extended, they return to point A while located between curves Ψ_{E^*} and K_2. When $n < 6/5$, curve 2, having reached the point E^*, passes into a curve which goes back into A between the curves K_1 and Ψ_{G^*}. Hence, it does not intersect curve V and the sign of the velocity does not change from negative to positive. Now, let $n > 6/5$. In this case, curve K_2, on arriving at E^* is subsequently continued by a curve which arrives at point A and is located between the curves K_1 and Ψ_{E^*}. It intersects curve V. Such a situation holds until the parameter n becomes equal to $3/2$. When $n = 3/2$, curve K_2 passes into a straight line $\Psi = -2F$ and coincides with Ψ_{E^*}. When $n > 3/2$, all of the curves which depart from point A between curves Ψ_{G^*} and K_2 when $F < 0$ arrive at the point E^* and are continued by curves which reach either point Q^* or B or A. In this case, they necessarily intersect the $F = 0$ axis, and we cannot satisfy the boundary condition (3.2).

The solvability of problem (1.1), (3.1), (3.2) is most conveniently investigated in the hodograph plane. The solution of Eq. (1.9), which satisfies condition (3.1), has the form

$$\psi_j = D_j\rho^j(1-z)^{1/3}F\left(\frac{1}{3}-j,\ \frac{1}{2}+j;\ \frac{4}{3};\ 1-z\right),\quad D_j > 0 \tag{3.3}$$

The analytic continuation of the solution (3.3) in the neighbourhood of $z = 0$ ($u < 0$, $\upsilon \to -0$) has the form [12]

$$\psi_j = D_j\rho^j(1-z)^{1/3}\left[\frac{\Gamma(4/3)\Gamma(1/2)}{\Gamma(1+j)\Gamma(5/6-j)}F\left(\frac{1}{3}-j,\frac{1}{2}+j;\frac{1}{2};z\right)+\right.$$
$$\left.+\frac{\Gamma(4/3)\Gamma(-1/2)}{\Gamma(1/3-j)\Gamma(1/2+j)}z^{1/2}F\left(1+j,\frac{5}{6}-j;\frac{3}{2};z\right)\right] \tag{3.4}$$

If $n > 6/5$ ($j < 5/6$), the first term on the right-hand side of (3.4) is positive. Passage through the $\upsilon = 0$ axis denotes intersection of curve V in the (F, Ψ)-phase plane.

Let us now establish for which values of n the curve in the (F, Ψ)-plane, corresponding to the solution (3.4), arrives at point A. Let us continue solution (3.4) through the $\upsilon = 0$ axis so that the corresponding solution is smooth in the physical plane. This condition will be satisfied if, in the domain $u < 0$, $\upsilon > 0$ the solution of Eq. (1.9) is taken in the form

$$\psi_j = D_j\rho^j(1-z)^{1/3}\left[\frac{\Gamma(4/3)\Gamma(1/2)}{\Gamma(1+j)\Gamma(5/6-j)}F\left(\frac{1}{3}-j,\frac{1}{2}+j;\frac{1}{2};z\right)-\right.$$
$$\left.-\frac{\Gamma(4/3)\Gamma(-1/2)}{\Gamma(1/3-j)\Gamma(1/2+j)}z^{1/2}F\left(1+j,\frac{5}{6}-j;\frac{3}{2};z\right)\right] \tag{3.5}$$

Analytic continuation of the solution (3.5) in the neighbourhood of $z = 1$ ($u = 0$, $\upsilon > 0$) yields

$$\psi_j = D_j\rho^j\left[\frac{2\pi\Gamma(1/3)\Gamma(4/3)}{\Gamma(1+j)\Gamma(1/2+j)\Gamma(5/6-j)\Gamma(1/3-j)}F\left(\frac{1}{6}+j,-j;\frac{2}{3};1-z\right)+\right.$$
$$\left.-\frac{2}{\sqrt{3}}\cos\pi\left(2j+\frac{1}{6}\right)(1-z)^{1/3}F\left(\frac{1}{3}-j,\frac{1}{2}+j;\frac{4}{3};1-z\right)\right] \tag{3.6}$$

When $6/5 < n < 3/2$ ($5/6 > j > 1/3$), the first term on the right-hand side of (3.6) is less than zero. This signifies that the stream function ψ_j vanishes at a certain z^*, and this means that, in the (F, Ψ)-plane, the integral curve arrives at A after intersecting curve V, and that all the requirements imposed on the solution of the problem are satisfied.

Let us now investigate the solvability of the problem when $n < 6/5$ in the hodograph plane. If $5/6 - j = -k$ $(k = 0, 1, \dots)$, the first term in (3.4) disappears, and we obtain the spectrum of the values

$$n_k = \frac{6k+6}{6k+5} (k = 0,\ 1,\ 2, \dots)$$

for which the stream function $\psi_j = 0$ when $u < 0, v = 0$.

The solution when $n_0 = 6/5$ $(k = 0)$ has been found in [5] and, in the phase plane, it is given by the functions $\Psi = \Psi^{(\pm)}(F)(F < 0)$ described above.

We will show that, for the remaining values of n_k, the solutions (1.8), (1.9) cannot be taken as solutions of the physical problem we have formulated.

We will represent Eq. (1.9) in the self-adjoint form [2]

$$\frac{d}{dz}\left(z^{1/2}(1-z)^{2/3}\frac{d\chi_j}{dz}\right) + \frac{j(j+1/6)}{z^{1/2}(1-z)^{1/3}}\chi_j = 0 \tag{3.7}$$

The coefficient in front of the second term on the right-hand side of (3.4) when $j - 5/6 = k$ is proportional to $\Gamma^{-1}(-k-1/2)$. If $k = 1$, it is negative. Consequently, the solution has just a single zero $z^* \in (0, 1)$. On the basis of Sturm's theorem [13] it may be asserted that all solutions (3.3) when $k > 1$ will have zeros. As a result, we obtain that the flow past a corner point with a rectilinear generatrix AO in the class of self-similar functions is described by the unique solution with $n = 6/5$.

We will now consider the case when the generatrix AO is curvilinear and $n < 6/5$ $(j > 5/6)$. It is seen from (3.4) that, if $0 < j - 5/6 < 1$, then, within the interval $0 < z < 1$, there is a zero since the first term in (3.4) is negative. Let us denote it by z_j^*. Then, when $1 < j - 5/6 < 2$, there is just a single zero of the solution (3.4) in the interval $(z_j^*, 1)$ according to Sturm's theorem. Reasoning in a similar manner, we arrive at the conclusion that, for all $j > 5/6$, the solution (3.3) has just a single zero in the interval (0, 1).

Hence, problem (1.1), (3.1), (3.2) has a solution in the class of self-similar functions when $6/5 \leqslant n < 3/2$.

4. We will now consider the problem of the transonic flow past a point of discontinuity with a rarefaction wave. It is required to find a smooth solution of the system of equations (1.1) which satisfies the no-flow condition (3.1) on the generatrix AO and which becomes a solution of the Prandtl–Mayer type when $\psi \to 0, x < 0$. The Vaglio-Laurin solution is represented in the (F, Ψ)-plane by an integral curve which departs from point A as curve K_1 and reaches E^*. From the point E^* it is extended by a curve of the family (2.3) and arrives at B. When $5/4 < n < 4/3$ $(2/3 > j > 1/2)$, curve K_1 arrives at the point E^* and is then continued by a curve which arrives at Q^* having first intersected the $F = 0$ axis. This means that there is a transition through the velocity of sound. One of the curves of the family (2.2), which departs from point A between K_1 and K_2 and is extended through E^*, arrives at point B. It necessarily intersects curve V which lead to a change in the sign of the velocity component v_y from positive to negative.

When $n = 4/3$ $(j = 1/2)$, curve K_1 becomes the straight line $\Psi = -3/2F$ and arrives at the point G^*.

When $4/3 < n$ $(j < 1/2)$, curve K_1 is located below the curve Ψ_{G^*}.

When $4/3 \leqslant n < 3/2$ $(1/2 \geqslant j > 1/3)$, one of the curves, which is located between Ψ_{G^*} and Ψ_{E^*}, has been continued through point E^*, it arrives at B. For these values of n, curve K_2 lies above the line $\Psi = -2F$ for all $F \in (-\infty, 0)$.

If $n = 3/2$ $(j = 1/3)$, curve K_2 becomes the straight line $\Psi_{E^*} = -2F$.

When $3/2 < n < 5/3$ $(1/3 > j > 1/4)$ one of the curves located between Ψ_{G^*} and K_2, after it has been extended through point E^*, arrives at B.

When n becomes equal to $5/3$, curve K_2 is extended at point E^* by a curve which arrives at B. In this case, all curves departing from point A between Ψ_{G^*} and K_2, after their continuation through point E^*, arrive at Q^* (Fig. 4).

When $n < 5/4$, all integral curves, located between K_1 and Ψ_{E^*} after being extended through E^*, return back to point A while being located below curve K_2. They cannot reach point B.

Let us consider the hodograph plane. In the neighbourhood of the limiting characteristic, the solution (1.9), which vanishes in it, has the form

$$\chi_j = z^{-j-1/6} F\left(\frac{1}{6}+j, \frac{2}{3}+j; \frac{7}{6}+2j; z^{-1}\right), \quad D_j > 0 \tag{3.8}$$

The solution (3.8) in the neighbourhood of the limiting characteristic corresponds, in the (F, Ψ)-plane, to the integral curve K_3. As a result of the analytic continuation of the solution (3.8) through the point $z = 1$ in the neighbourhood of the point $z = 0$ ($u < 0$, $\upsilon = 0$) we find

$$\psi_j = D_j\rho^j \left[2\sqrt{\pi}\frac{\Gamma(7/6+2j)\sin\pi(1/3+j)}{\Gamma(2/3+j)\Gamma(1+j)} F\left(\frac{1}{3}-j, \frac{1}{2}+j; \frac{1}{2}; z\right) - \right.$$
$$\left. -4\sqrt{\pi}\frac{\Gamma(7/6+2j)\cos\pi(1/3+j)}{\Gamma(1/2+j)\Gamma(1/6+j)} z^{1/2} F\left(1+j, \frac{5}{6}-j; \frac{3}{2}; z\right)\right] \tag{3.9}$$

If

$$1/3 + j = k \quad (k = 1, 2, \ldots) \tag{3.10}$$

then the stream function (3.9) vanishes when $\upsilon = 0$. This situation corresponds to the flow past a corner point with a rectilinear generatrix AO. We obtain the Vaglio-Laurin solution when $k = 1$.

We will show that the remaining values of the spectrum (3.10) do not yield physically meaningful solutions. The solution (3.9), subject to condition (3.10), takes the form

$$\psi_j = -D_j\rho^j \left[4\sqrt{\pi}\frac{\Gamma(1/2+2k)\cos\pi k}{\Gamma(1/6+k)\Gamma(-1/6+k)} z^{1/2} F\left(\frac{2}{3}+k, \frac{7}{6}-k; \frac{3}{2}; z\right)\right] \tag{3.11}$$

When $k = 2m$, the value of ψ_j (3.11) is less than zero when $z \to 0$, $\upsilon < 0$. When $z = 1$, the solution (3.8) takes the value

$$\psi_j = D_j\rho^j \frac{\Gamma(1/3)\Gamma(1/2+2k)}{\Gamma(2/3+k)\Gamma(1/6+k)} > 0$$

Consequently, the solution (3.8) vanishes for even k in the interval [0, 1]. Since the solution (3.8) satisfies Eq. (3.7), Sturm's theorem can be applied and it follows from this that, when $k = 2m + 1$ and $m \geqslant 1$, the solution (3.8) also vanishes.

We will first consider the case when $n > 5/4$. Continuation of the solution (3.9) in the neighbourhood of the line $z = 1$ in the domain $\upsilon > 0$ yields

$$\psi_j = 2D_j\rho^j \left[\frac{\Gamma(7/6+2j)\Gamma(1/3)}{\Gamma(1+j)\Gamma(1/2+j)} \cos 2\pi j F\left(\frac{1}{6}+j, -j; \frac{2}{3}; 1-z\right) + \right.$$
$$\left. + \frac{\Gamma(7/6+2j)\Gamma(-1/3)}{\Gamma(2/3+j)\Gamma(1/6+j)} \cos\pi(1/3+2j)(1-z)^{1/3} F\left(\frac{1}{3}-j, \frac{1}{2}+j; \frac{4}{3}; 1-z\right)\right] \tag{3.12}$$

The first term in the solution (3.12) is negative when $5/4 < n < 5/3$ ($1/4 < j < 2/3$). Consequently, the stream function vanishes within the interval $0 < z < 1$ ($u < 0$, $\upsilon > 0$). The value of the root z^* enables us to find the relation between the constants A_0 and B_0. So, the problem has a solution when $5/4 < n < 5/3$.

Let us now consider the case when $6/5 < n < 5/4$ ($2/3 < j < 5/6$). For these values of j, the first term on the right-hand side of (3.9) is negative. This means that the stream function ψ_j vanishes within the interval $0 < z < 1$ ($u < 0$, $\upsilon < 0$). This indicates that the integral curve reaches point A without intersecting the line V. The case $n < 6/5$ ($j > 5/6$) is similar to that considered earlier.

Hence, there are two families of self-similar solutions of Karman's equation. One of them, when $^6/_5 < n < ^3/_2$, describes the transonic flow past a corner point on a profile with a free stream-line which is formed mainly under the influence of the curvature of the generatrix AO. The other family, when $^5/_4 < n \leqslant ^5/_4$, describes such a process with a rarefaction wave. These solutions exist in parallel with solutions which can be constructed on the basis of the solutions when $n = ^6/_5$ and $n = ^5/_4$ by the addition of terms which take account of the curvature. It is of interest to find which of the local solutions described above are realized in practice. This is not simple since not only the occurrence of corner points but also the boundary conditions affect the choice of solutions. It has been shown [11] that, under the influence of the boundary conditions quite another type of flow past a corner point can be generated and realized when $n = 2$. In answering the above question it is necessary firstly to clarify the role played by viscosity and heat conduction in the neighbourhood of the corner point on the profile. Although the problem of the uniqueness of Vaglio-Laurin solution was raised a long time ago [9], it, supplemented by the question of the uniqueness of the solution when $n = ^6/_5$, remains open. In all likelihood, the choice of solution depends on the global problem under consideration. The solution which is realized in it is due to the pressure gradient which is generated in the neighbourhood of the corner point.

Note. In my paper "Investigation of self-similar solutions describing a flow in mixing layers" (*Prikl. Mat. Mekh.* **50,** 3, 403–414, 1986) the sixth line on page 412 after the formulae should read: "Hence, for $m > ^1/_2$ and a specified value of $b > 0$, the solution of problem (2)–(5) exists in the class of triply continuous differentiable functions and is unique when $m \geqslant 1$."

The error which slipped in is immaterial since, in the range of values $m \in (^1/_2, 1)$, problems having a physical meaning are unknown. When $m \in (^1/_2, 1)$, (as also for all $m > ^1/_2$), the existence and uniqueness of the physical problem corresponding to (2)–(5) is ensured as was noted at the beginning of the paper, for example, by the conditions: $\Phi(\zeta) \to C < 0$, $\varsigma \to -\infty$; $\Phi'(\varsigma) > 0$, $\varsigma \in (-\infty, +\infty)$. The second condition can also be replaced by the requirement $\Phi(\varsigma) \to +\infty, \varsigma \to +\infty$.

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